Microwaves

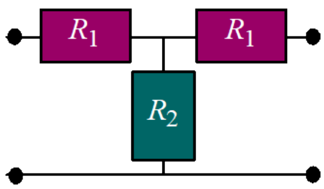
Series 8, solutions

Problem 1

We want to realize a T attenuator which is variable, matched to a 50Ω line and producing an attenuation between 3 and 10 dB. Determine the range of value of the variable resistors we will need to use

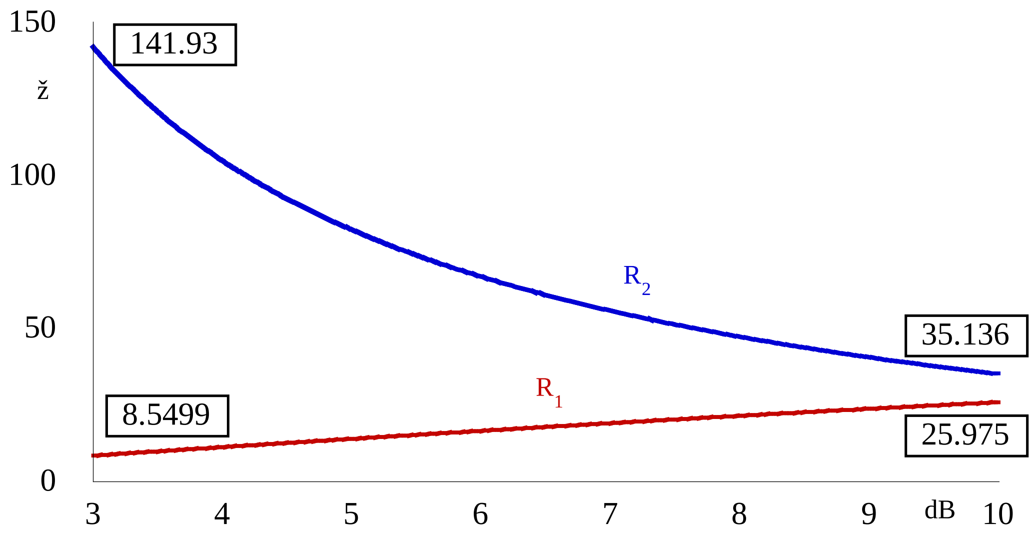
$$A(\mathrm{dB}) = -20\log_{10}|S_{21}| = 20\log_{10}\frac{Z_c + R_1}{Z_c - R_1} \quad \text{and thus} \quad \frac{Z_c + R_1}{Z_c - R_1} = 10^{A(dB)/20}$$

Thus $R_1 = Z_c \dfrac{10^{A(dB)/20} - 1}{10^{A(dB)/20} + 1} = 50\dfrac{10^{A(dB)/20} - 1}{10^{A(dB)/20} + 1}\ \Omega$



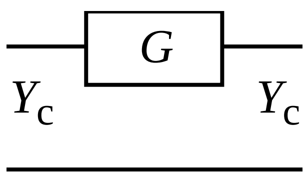
$$R_2 = \frac{Z_c^2 - R_1^2}{2R_1} = \frac{50^2 - \left(50\dfrac{10^{A(dB)/20} - 1}{10^{A(dB)/20} + 1}\right)^2}{2\cdot 50\dfrac{10^{A(dB)/20} - 1}{10^{A(dB)/20} + 1}} = 100\frac{10^{A(dB)/20}}{\left(10^{2A(dB)/20} - 1\right)} = \frac{100}{\left(10^{A(dB)/20} - 10^{-A(dB)/20}\right)}\Omega$$

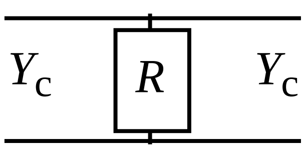
These curves are shown in the following figure, where the limit values for the resistors are also shown



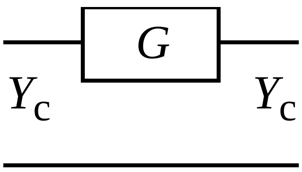
Problem 2

A switch is connected on a 50 Ω transmission line. When the switch is open, it can be represented as a 10'000 Ω resistor, when it is closed as a 1 Ω resistor. Find the scattering matrix for this open and closed switch, considering first that the switch is in series on the line and then that it is on parallel on the line (see figure) . Compare with the "ideal" case $R = 0$ and $R = \infty$.





We want to find the scattering matrix of a resistor, connected in series or in parallel on a line. In the case of a series connection, we have



$$[\underline{Y}] = \begin{bmatrix} G & -G \\ -G & G \end{bmatrix} \qquad [\underline{S}] = \frac{1}{Y_c + 2G}\begin{bmatrix} Y_c & 2G \\ 2G & Y_c \end{bmatrix}$$

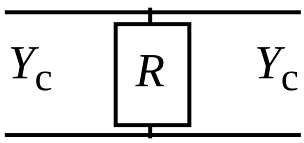
With the two limiting cases

open switch (10 kΩ) $[\underline{S}] = \begin{bmatrix} 0{,}99 & 0{,}01 \\ 0{,}01 & 0{,}99 \end{bmatrix}$ attenuation 40,1 dB

ideal open circuit $[\underline{S}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ attenuation ∞ dB closed switch (1 Ω) $\quad [\underline{S}] = \begin{bmatrix} 0,01 & 0,99 \\ 0,99 & 0,01 \end{bmatrix}$ attenuation 0,086 dB

ideal case $\quad [\underline{S}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ attenuation 0 dB

In the case of a parallel connection, we get



$$[\underline{Z}] = \begin{bmatrix} R & R \\ R & R \end{bmatrix} \qquad [\underline{S}] = \frac{1}{Z_c + 2R} \begin{bmatrix} -Z_c & 2R \\ 2R & -Z_c \end{bmatrix}$$

with the following limiting cases

open switch $\quad [\underline{S}] = \begin{bmatrix} -0,0025 & 0,9975 \\ 0,9975 & -0,0025 \end{bmatrix}$ attenuation 0,022 dB

"ideal case" $\quad [\underline{S}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ attenuation 0 dB

closed switch $\quad [\underline{S}] = \begin{bmatrix} -0,962 & 0,038 \\ 0,038 & -0,962 \end{bmatrix}$ attenuation 28,3 dB

ideal case : short circuit $\quad [\underline{S}] = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ attenuation ∞ dB

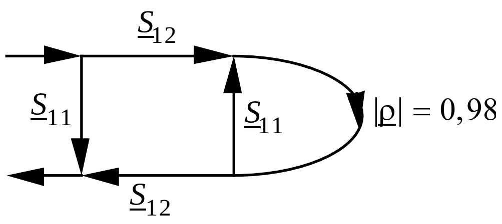
We note that for this case, the two configurations are not equivalent: the series connection given a better isolation in the open case, while the parallel configuration has less insertion losses.

Problem 3

We want to measure the attenuation of a reciprocal and symmetric two port device. To this aim, we terminate port two of the device with a short circuit, having a reflection coefficient $|\underline{\rho}|$ equal to 0,98 (the phase is not known).

At port one of the device, we measure then a VSWR of 3.5.

A second measurement is done, but this time with port two terminated by a perfect matched load ($\underline{\rho}$ = 0), and we obtain a VSWR of 1.3 at port one of the device. What can we say about the attenuation of the device (s_{21}) ? What can we say about the error margins of the measurements ?



When the two port is terminated by a matched load, the VSWR is equal to 1.3, which yields directly the amplitude of the reflection coefficient at port one $\left|\underline{S}_{11}\right| = (1,3-1)/(1,3+1) = 0,1304$.

The information relative to the two port terminated by a short circuit (see figure), hives us

$$\underline{\rho}_{in} = \underline{S}_{11} + \frac{0,98 \cdot \underline{S}_{12}^2}{1-0,98 \cdot \underline{S}_{11}} \qquad \text{and} \qquad \left|\underline{\rho}_{in}\right| = \frac{3,5-1}{3,5+1} = 0,55555$$

We deduce : $\underline{S}_{12} = \sqrt{\dfrac{\left(\underline{\rho}_{in} - \underline{S}_{11}\right)\left(1-0,98\cdot\underline{S}_{11}\right)}{0,98}}$.

Unfortunately, we do not know the phases of S_{11} and ρin, so we cannot find even the amplitude of S_{12} exactly. We can only say that it lies between the following limits :

$$0,615 = \sqrt{\frac{(0,55555-0,1304)(1-0,98\cdot 0,1304)}{0,98}} \le \left|\underline{S}_{12}\right|$$

$$\le \sqrt{\frac{(0,55555+0,1304)(1+0,98\cdot 0,1304)}{0,98}} = 0,8884$$

These two values correspond to an attenuation of 4.222 dB and 1.027 dB respectively. We can thus say that the two port produces an attenuation of 2,625 ± 1,598 dB, and that the error margin is of ± 61 %.

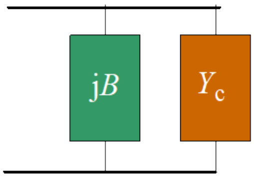
Problem 4

An inductive lossless iris is placed in a waveguide, producing an attenuation of 1.5 dB. An iris is a very thin obstacle placed transversally in the waveguide. Determine its normalized susceptance B/Yc (which will of course be inductive), the amplitude of the scattering parameters and the measured input VSWR. We suppose that the waveguide is terminated by a matched load.

An attenuation of 1,5 dB corresponds to $\left|\underline{S}_{21}\right| = 10^{-1,5/20} = 0,8414 \cong \sqrt[4]{0,5}$.

As the iris is lossless, we have $\left|\underline{S}_{11}\right| = \sqrt{1-\left|\underline{S}_{21}\right|^2} = \sqrt{1-0,8414^2} = 0,5404$

The equivalent circuit of the obstacle with the matched terminations at the second port is



$$\mathrm{ROS} = \frac{1 + 0,5404}{1 - 0,5404} = 3,35$$

The input reflection coefficient is given by

$$\underline{\rho} = \underline{S}_{11} = \frac{Y_c - (Y_c + \mathrm{j}B)}{Y_c + (Y_c + \mathrm{j}B)} = \frac{-\mathrm{j}B}{2Y_c + \mathrm{j}B}$$

and $|\underline{\rho}|^2 = \dfrac{(B/Y_c)^2}{4 + (B/Y_c)^2} = 0,5404^2 = 0,292$

We get finally that $(B/Y_c) = -2\sqrt{\dfrac{|\underline{\rho}|^2}{1 - |\underline{\rho}|^2}} = -2\dfrac{|\underline{S}_{11}|}{|\underline{S}_{21}|} = -2\dfrac{0,5404}{0,8414} = -1,2845$